Design Guidelines for Training-based MIMO Systems with Feedback

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Overview

- Background:
  - MIMO systems
  - Training based transmission

- The design problems

- Information capacity

- Results
  - Non-feedback system
  - Channel gain feedback (CGF) system

- Conclusions
Background: MIMO Systems

\[ y = Hx + n \]
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- Non-feedback systems:
  - No information about the channel is available at TX.
Background: MIMO Systems

- Non-feedback systems:
  - No information about the channel is available at TX.

- Channel gain feedback (CGF) systems:
  - The estimated channel gains are known at the TX.
Background: Channel Model

- $N_t$ transmit antennas and $N_r$ receive antennas
- Channel is constant during one transmission block

$$y = Hx + n$$
Background: PSAM

- **Pilot-symbol-assisted modulation (PSAM):**
  - Insert pilots (known at the receiver) into data transmission to facilitate channel estimation at receiver.

\[
y = H x_p + n \xrightarrow{\text{guess}} \hat{H}
\]
Background: PSAM

- Pilot-symbol-assisted modulation (PSAM)

\[ P = (1-\alpha)P + \alpha P \]

- \( \alpha = \text{PSAM power factor (fraction of power allocated to data)} \)
Background: PSAM

- Pilot-symbol-assisted modulation (PSAM)

$$P = (1-\alpha)P + \alpha P$$

<table>
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<tr>
<th>Pilot</th>
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<tbody>
<tr>
<td>$L_p$</td>
<td>$L_d$</td>
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- $\alpha = \text{PSAM power factor (fraction of power allocated to data)}$
- $L_p = \text{training length (amount of time allocated to pilots)}$
The Design Problems

- What is the optimal transmission design which maximizes the information capacity of the MIMO system?

- information capacity: the number of bits per second that can be transmitted.
A capacity lower bound per transmission block

\[ \overline{C}_{LB} = \frac{L - L_p}{L} \mathbb{E}_{\hat{H}} \left\{ \log_2 \left| I_{N_t} + \left( \sigma_n^2 + \text{tr}\{R_{\hat{H}}Q\} \right)^{-1} \hat{H}^\dagger \hat{H}Q \right| \right\} \]

where \( Q = \mathbb{E}\{xx^\dagger\} \)

and \( R_{\hat{H}} = \mathbb{E}\{\hat{H}^\dagger \hat{H}\}/N_r = (R_H^{-1} + \frac{1}{\sigma_n^2} X_p X_p^\dagger)^{-1} \)

Is the lower bound accurate? Yes.

What are the design parameters?
Capacity Lower Bound

- A capacity lower bound per transmission block

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\overline{C}_{LB} = \frac{L - L_p}{L} E_{\hat{H}} \left\{ \log_2 \left| I_{N_t} + \left( \sigma_n^2 + \text{tr}\{ R_{\hat{H}} Q \} \right)^{-1} \hat{H}^\dagger \hat{H} Q \right| \right\}
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- Pilot parameters:
  - Spatial structure of pilot symbols, i.e., \( X_p \)
Capacity Lower Bound

A capacity lower bound per transmission block

$$\overline{C}_{LB} = \frac{L - L_p}{L} E_{\hat{H}} \left\{ \log_2 \left| I_{N_t} + \left( \sigma_n^2 + \text{tr} \{ R_{\tilde{H}} Q \} \right)^{-1} \hat{H}^\dagger \hat{H} Q \right| \right\}$$

where $Q = E\{xx^\dagger\}$

and $R_{\tilde{H}} = E\{\tilde{H}^\dagger \tilde{H}\}/N_r = (R_{\tilde{H}}^{-1} + \frac{1}{\sigma_n^2} X_p X_p^\dagger)^{-1}$

Pilot parameters:

- The number of pilot symbols or training length, i.e., $L_p$
Capacity Lower Bound

- A capacity lower bound per transmission block

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- Pilot parameters:
  - Temporal power allocation to pilot and data, i.e., the PSAM power factor, \( \alpha \)
Capacity Lower Bound

- A capacity lower bound per transmission block

\[
\overline{C}_{LB} = \frac{L - L_p}{L} E_{\hat{H}} \left\{ \log_2 \left| I_{N_t} + \left( \sigma_n^2 + \text{tr}\{R_{\hat{H}}Q\} \right)^{-1} \hat{H}^\dagger \hat{H}Q \right| \right\}
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- Data parameters:
  - Spatial correlation of data symbols \( Q \)
  - Power allocation for data transmission
The Design Parameters

- **Goal:** To maximize the information capacity.

- **Design parameters**
  - Pilot spatial structure, i.e., $X_p$
  - Data spatial structure, i.e., $Q$
  - PSAM power factor (power allocation to pilot and data), $\alpha$
  - Training length, i.e., $L_p$

- Different design solutions for different systems:
  - Non-feedback and channel gain feedback.
The Design Parameters

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- Different design solutions for different systems:
  Non-feedback, and channel gain feedback.
Non-feedback System with Spatially i.i.d. Channels

Non-feedback Systems:

All 4 design parameters have closed-form solutions and can be found in the existing literature.
Channel Gain Feedback (CGF)
Systems with i.i.d. channels

- Does the optimal design for non-feedback systems still works well for channel gain feedback systems?
- What happens if there is a delay in the feedback?
- Design parameters: spatial structure of pilot and data, $X_p$ and $Q$, PSAM power factor, $\alpha$, training length, $L_p$. 
Channel Gain Feedback (CGF)

Systems with i.i.d. channels

- Ideal case: no feedback delay

- Optimal pilot spatial structure: orthogonal with equal power, i.e.,
  \[ X_p X_p^\dagger = \frac{P_p L_p}{N_t} I_{N_t} \]
Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Ideal case: no feedback delay

- Optimal pilot spatial structure: orthogonal with equal power, i.e.,
  \[ X_p X_p^\dagger = \frac{\mathcal{P}_p L_p}{N_t} I_{N_t} \]

- Optimal data transmission, \( Q \): water-filling according to \( \hat{H}^\dagger \hat{H} \)
Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Ideal case: no feedback delay

- The optimal solutions for
  - PSAM power factor $\alpha^*$, and,
  - training length $L_p^*$

  coincide with those for the non-feedback systems.

- It is good news: the same design is optimal for both non-feedback and feedback systems.
Channel Gain Feedback (CGF)  

Systems with i.i.d. channels

- Practical case: with feedback delay
Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Practical case: with feedback delay

| Pilot | Non-adaptive data sub-block | Adaptive data sub-block |
Channel Gain Feedback (CGF) 
Systems with i.i.d. channels

- Practical case: with feedback delay

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\[ d = \beta L_d \]

- \( \beta \) is the feedback delay factor, \( \beta = d / L_d \)

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Pilot

Non-adaptive data sub-block

Adaptive data sub-block

\[ L_d \]

\[ L_d - d \]
Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Practical case: with feedback delay

- $\beta$ is the feedback delay factor, $\beta = \frac{d}{L_d}$
- $\phi$ is the ratio of power allocated to the non-adaptive data sub-block.
Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Practical case: with feedback delay

- We have an additional design parameter: temporal power division among the non-adaptive and the adaptive data sub-blocks, i.e., $\phi$.

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Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Practical case: with feedback delay

- We have an additional design parameter: temporal power division among the non-adaptive and the adaptive data sub-blocks, i.e., $\phi$.

| Pilot | Non-adaptive data sub-block | Adaptive data sub-block |

- A sub-optimal solution: $\phi=\beta$, i.e., transmit equal power per data transmission.
Channel Gain Feedback (CGF)
Systems with i.i.d. channels

- Practical case: with feedback delay

- We have an additional design parameter: temporal power division among the non-adaptive and the adaptive data sub-blocks, i.e., $\phi$.

A sub-optimal solution: $\phi = \beta$, i.e., transmit equal power per data transmission.
Capacity vs. SNR
showing optimality of using $\phi = \beta = 0.208$
Capacity vs. SNR showing optimality of using $\phi = \beta = 0.208$
Capacity vs. SNR showing optimality of using $\phi = \beta = 0.208$

Lines: use $\phi = \beta$

Markers: numerically search for the optimal $\phi$
Channel Gain Feedback (CGF) Systems with i.i.d. channels

- Practical case: with feedback delay

- With $\phi=\beta$, the optimal solutions for
  - PSAM power factor $\alpha^*$, and,
  - training length $L_p^*$ coincide with those for the delayless systems.

- It is good news: the same design is optimal for both
  - delayless feedback systems, and
  - delayed feedback systems.
Conclusions

- We have studied the optimal training resource allocation in terms of
  - Power allocation, and
  - Time allocation
  between channel estimation and data transmission.

- We have found that
  - The optimal solution for non-feedback systems is still near optimal for channel gain feedback systems, and
  - The feedback delay has little effect on this optimal solution.
Thank you for your attention!
Q? or BBQ?