Implementation of Classical Linear Stochastic Systems Using Linear Quantum Components

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1 Introduction
Measurement feedback control of quantum systems is important in a number of areas of quantum technology, including quantum optical systems, nanomechanical systems, and circuit QED systems. In measurement feedback control, the plant is a quantum system, while the controller is a classical (i.e. non-quantum) system. The classical controller processes the outcomes of a measurement of an observable of the quantum system (e.g. the quadrature of an optical field) to determine the classical control actions that are applied to control the behavior of the quantum system. The closed loop system involves both quantum and classical components, such as an electronic device for measuring a quantum signal, as shown in Figure 1.

![Figure 1: Measurement feedback control of a quantum system.](image)

2 Motivation
The advantages of implementation of the classical controller using quantum components are as follows:

1. Faster response and processing times
2. Becoming feasible to implement quantum networks in semiconductor materials.
3. Implementing control networks on the same chip (rather than interfacing to a separate system)

The purpose of this poster is to show how classical linear stochastic systems can be physically implemented using quantum optical components as shown in Figure 2.

![Figure 2: Quantum realization of a measurement feedback control system.](image)

3 Problem Formulation
3.1 What is the relation between classical and quantum systems?
Specifically, we consider a class of classical linear stochastic systems of the form

\[ \dot{x}(t) = A \dot{x}(t) + B \dot{u}(t) + B \ddot{w}(t), \]

where \( \dot{x}(t) \) and \( \ddot{w}(t) \) satisfy classical stochastic processes. The system has a transfer function \( \Xi(s) \) from the noise input channel \( v \) to the output channel \( y \) given by.

\[ \Xi(s) = C(sI - A)^{-1}B + I_{n_y} \]

where \( I_{n_y} \) is an identity matrix of order \( n_y \).

We also consider a class of quantum linear stochastic systems of the form

\[ \dot{x}(t) = \tilde{A} \dot{x}(t) + B \dot{u}(t) + B \ddot{w}(t), \]

where \( \dot{x}(t) \) is a vector of self-adjoint possibly non-commuting operators, with the initial value \( x(0) = x_0 \) satisfying the commutation relations \( x_0 \dot{x}_0 = \dot{x}_0 x_0 = 2 \theta \dot{x}_0 x_0 \).

where \( \theta \) is a skew-symmetric real matrix. The matrix \( \theta \) is said to be canonical if it is the form \( \theta = \theta_0 \). The components of the vector \( \theta \) are quantum stochastic processes with the following non-zero lift functions,

\[ \xi_j(t) = \theta_j(t), \quad j = 1, \ldots, n_y \]

where \( \theta_j(t) \) is a quantum stochastic process.

3.2 Quantum Physical Realization
The quantum system (3) is canonically physically realizable, if and only if the matrices \( \tilde{A}, \tilde{B}, \tilde{C} \) and \( \tilde{D} \) satisfy the following conditions:

\[ A \tilde{A} + B \tilde{B} + B \tilde{D} \tilde{D} = 0, \quad \tilde{B} = \tilde{D} \tilde{D} \tilde{B} \quad \tilde{D} = \tilde{D} \tilde{D} \tilde{B} \]

3.3 Measurement Feedback Control of Classical Linear Stochastic Systems
In the case of classical linear stochastic systems, one can realize the above classical system, with the following matrices,

\[ A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad D = 1 \]

5 An Example
By the above theorem, we can construct a quantum system \( \tilde{C} \), an augmentation of the above classical system, with the following matrices,

\[ \tilde{A} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \tilde{D} = 1 \]

6 Conclusion
In this poster, we have shown that a class of classical linear stochastic systems (having a certain form and satisfying certain technical assumptions) can be realized by quantum linear stochastic systems. For this class of classical systems, we have derived results that show how to explicitly construct the quantum realization of a given classical system of this class. Our results are illustrated with quantum realization examples from quantum optics. It is hoped that the main results of the work will aid in facilitating the implementation of classical linear systems with fast quantum optical devices, especially in miniature platforms like nanophotonic circuits.

References