Models of Robustness in Temporal Planning and Scheduling

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Motivation
- Explore a variety of robustness measures
- Figure out what exactly these metrics measure
- Find relationships among the metrics
* Give a better way to measure robustness

Background
1. Partial Order Schedules (Policella 2004)
   - A consistent Simple Temporal Network
   - Defined and added time constraints

2. Robustness Measures
   a. Flexibility (Aloulou and Portmann 2003)
      - the fraction of pairs with no relation
      \[
      \text{flex} = \frac{|\{(a, a')| a_i \neq a_i \wedge a_j = a_j\}|}{n(n - 1)}
      \]
      - the degree of interaction
   b. Fluidity (Cesta, Oddi and Smith 1998)
      - the average slack between pairs of activities
      \[
      \text{fldt} = \sum_{i=1}^{n} \sum_{j=1/j \neq i} \frac{\text{slack}(a_i, a_j)}{H \times n \times (n - 1)} \times 100
      \]
      - the ability to absorb temporal variation
   c. Robust Makespan (Na and Fei 2012)
      - Durational Uncertainty: \(d_i = d_i + \tilde{z}_i\)
      - Robust Makespan: \(P(\text{makespan} \leq F^*) \geq 1 - \epsilon\)
      - Probabilistic model with approximation

3. STNU and Dynamic Control
   a. STN with Uncertainty (Vidal 1999)
      - Requirement and Contingent Links
   b. Dynamic Control
      - Checking dynamic controllability for a STNU is strongly polynomial (Morris, Muscettola and Vidal, 2001 and 2005)
      - The problem of optimizing a function of link bounds, subject to dynamic controllability is NP-hard (Wah and Xin, 2006)

Constraint Model of Dynamic Controllability

Introduced by Wah and Xin 2004

Shortest Path

\[
\begin{align*}
\text{if } & iBC \geq 0 \\
\text{then } & iAC \leq u_{AB} + l_{BC} \leq u_{AC} \\
\text{if } & iBC \geq 0 \\
\text{then } & iAB \leq u_{AB} - l_{BC} \\
\text{if } & iAB \geq 0 \\
\text{then } & iAC \leq u_{AB} + l_{BC} \leq u_{AC} \\
\end{align*}
\]

Precedence and Waits

\[
\begin{align*}
\text{if } & iBC \geq 0 \\
\text{then } & w_{ABC} \geq w_{AB} - u_{DB} \\
\text{if } & w_{ABC} \geq 0 \\
\text{then } & w_{ADC} \geq w_{ABC} - i_{BC}^{R/C} \\
\end{align*}
\]

Waits Regression

\[
\begin{align*}
\text{if } & w_{ABC} \geq 0 \\
\text{then } & w_{ADC} \geq w_{ABC} - u_{DB} \\
\end{align*}
\]

Waits Bounds

\[
\begin{align*}
\text{if } & w_{ABC} \geq 0 \\
\text{then } & w = i^R \leq l^C \\
\end{align*}
\]

The MIP model =>

Maximum Delay with Dynamic Control

1. A new worst-case robustness measure
2. The temporal tolerance is represented by the shortest maximal delay

\[
\text{MD} = \max(Z)
\]

s.t. \(l(e) = \text{duration}(e)\)

\[
\text{u}(e) = l(e) + z(e)\]

\[
Z \leq z(e)\]

\[
L(e) \leq l(e) \leq u(e) \leq U(e)\]

DC constraints

Results

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<tr>
<th>STNU</th>
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