Models of Robustness in Temporal Planning and Scheduling

Jing Cui – PhD Student
cui.jing@anu.edu.au
Supervisors: Dr. Patrik Haslum, Prof. Sylvie Thiebaux and Dr. Hassan Hijazi

NICTA

Motivation

- Explore a variety of robustness measures
- Figure out what exactly these metrics measure
- Find relationships among the metrics

* Give a better way to measure robustness

Background

1. **Partial Order Schedules (Policella 2004)**
   - A consistent Simple Temporal Network
   - Defined and added time constraints

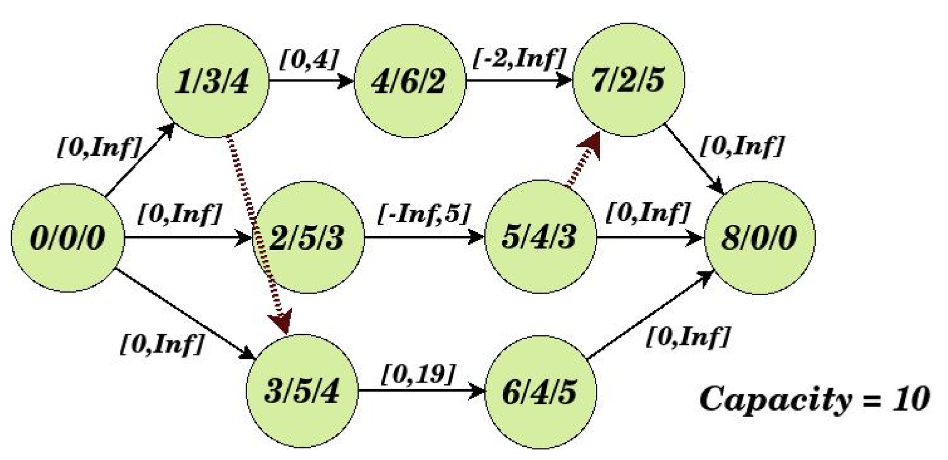


Figure from Na and Fei 2012

2. **Robustness Measures**

a. Flexibility (Aloulou and Portmann 2003)

- the fraction of pairs with no relation

$$flex = \frac{|\{(a_i, a_j) | a_i \not\prec a_j \wedge a_j \not\prec a_i\}|}{n(n-1)}$$

- the degree of interaction

b. Fluidity (Cesta, Oddi and Smith 1998)

- the average slack between pairs of activities

$$fldt = \sum_{i=1}^{n} \sum_{j=1 \wedge j \neq i}^{n} \frac{slack(a_i, a_j)}{H \times n \times (n-1)} \times 100$$

- the ability to absorb temporal variation

c. Robust Makespan (Na and Fei 2012)

- Durational Uncertainty: $\bar{d}_i = d_i^0 + \tilde{z}_i$
- Robust Makespan: $P(makespan \leq F^*) \geq 1 - \epsilon$
- Probabilistic model with approximation

3. **STNU and Dynamic Control**

a. STN with Uncertainty (Vidal 1999)

- Requirement and Contingent Links

b. Dynamic Control

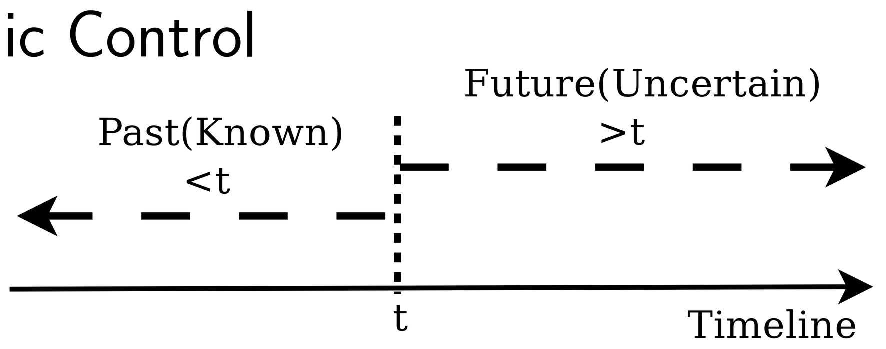


Figure from Morris et al. 2001

- Checking dynamic controllability for a STNU is **strongly polynomial** (Morris, Muscettola and Vidal,2001 and 2005)
- The problem of optimizing a function of link bounds, subject to dynamic controllability is **NP-hard**(Wah and Xin, 2006)

Constraint Model of Dynamic Controllability

Introduced by Wah and Xin 2004

Shortest Path

$$\begin{cases} l_{AC} \leq u_{AB} + l_{BC} \leq u_{AC} \\ l_{AC} \leq l_{AB} + u_{BC} \leq u_{AC} \\ u_{AB} + u_{BC} \geq u_{AC} \\ l_{AB} + l_{BC} \leq l_{AC} \end{cases}$$

Precedence and Waits

if $l_{BC} \geq 0$

$$\begin{cases} l_{AB} \geq u_{AC} - u_{BC} \\ u_{AB} \leq l_{AC} - l_{BC} \end{cases}$$

$$w_{ABC} \geq u_{AC} - u_{BC}$$

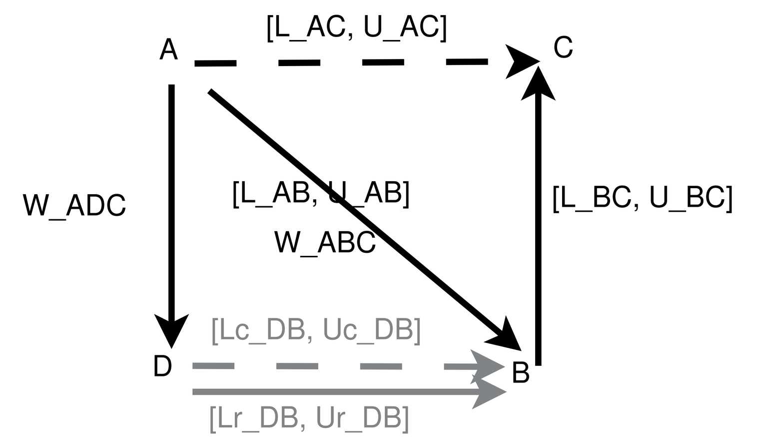


Figure from Morris et al. 2001

Waits Regression

$$w_{ADC} \geq w_{ABC} - u_{DB}^{R/C}$$

if $w_{ABC} \geq 0$

$$w_{ADC} \geq w_{ABC} - l_{DB}^{C}$$

Waits Bounds

$$\begin{cases} w = l^R \quad U^R \leq L^C \\ l^R \geq l^C \quad w \geq l^C \\ l^R > w \quad w < l^C \end{cases}$$

The MIP model =>

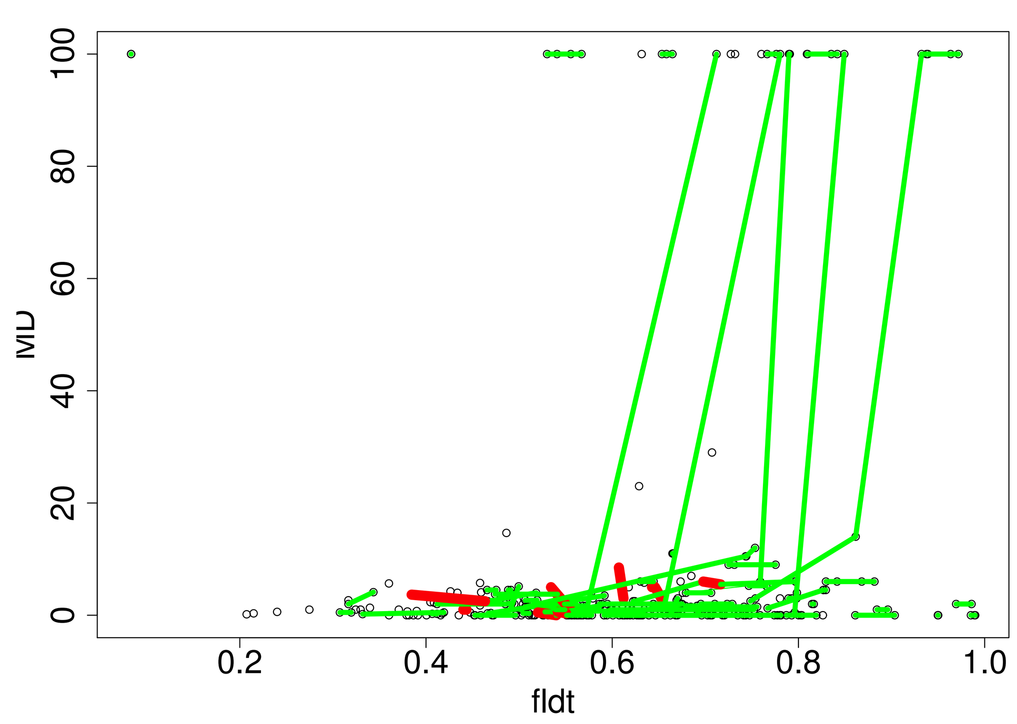
$$\begin{cases} w_1 - x(U_{AB} + 1) < 0 \\ w_1 - (1 - x)L_{AB} \geq 0 \\ w_2 - w_1 + l_{DB}^C \geq (1 - x)K \\ w_2 - w_1 + u_{DB}^C - xK \geq 0 \\ K = L_{AD} - U_{AB} + L_{DB} \end{cases}$$

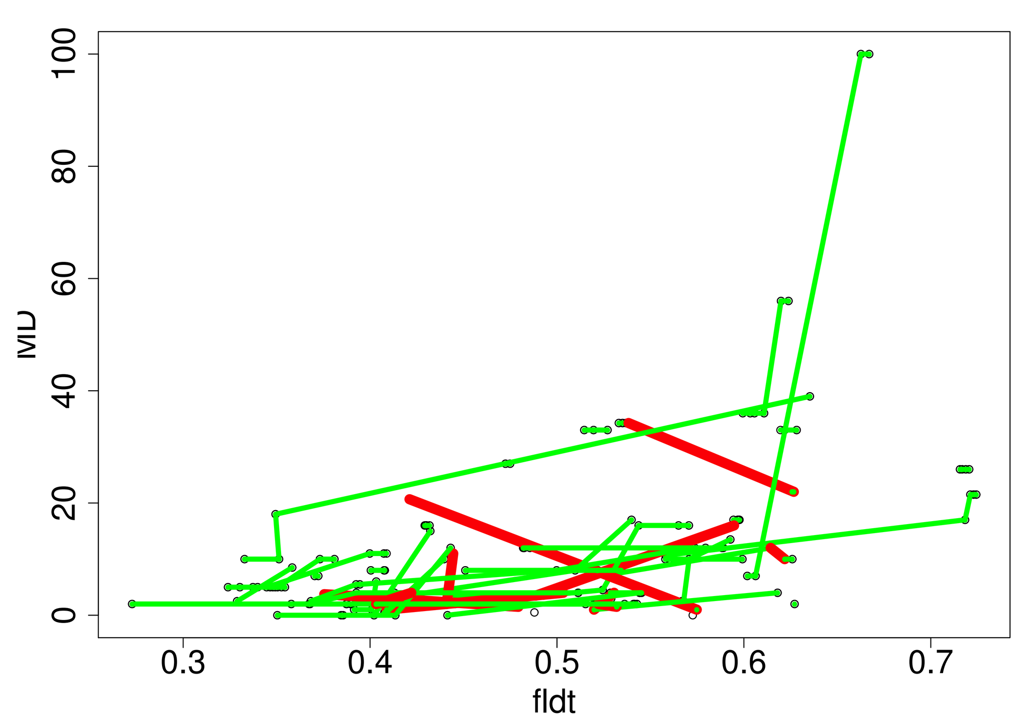
Maximum Delay with Dynamic Control

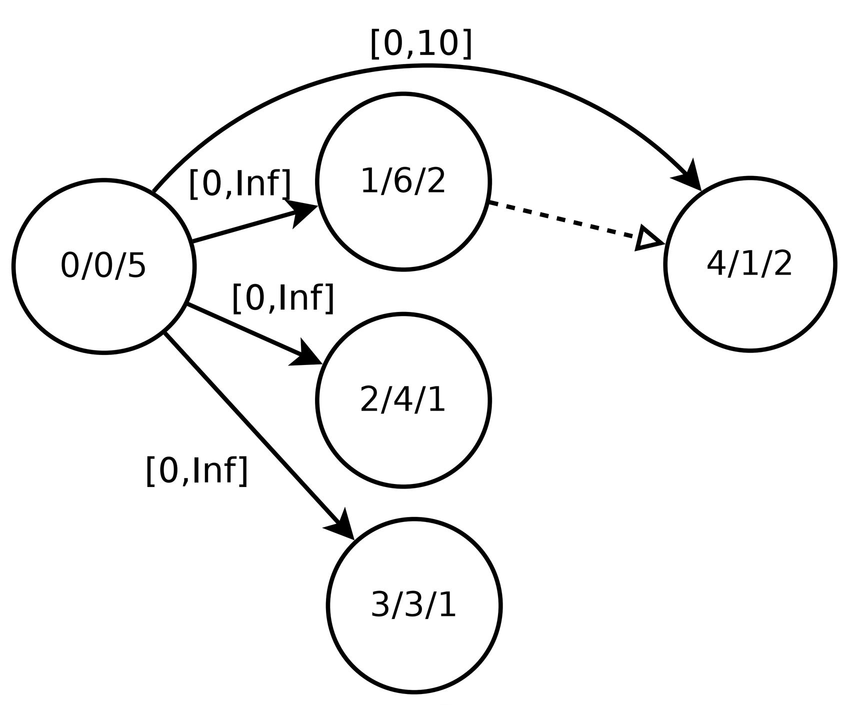
1. A new worst-case robustness measure
2. The temporal tolerance is represented by the shortest maximal delay

$$\begin{aligned} & MD = max(Z) & \\ s.t.\ & l(e) = duration(e) & e \in C \\ & u(e) = l(e) + z(e) & e \in C \\ & Z \leq z(e) & e \in C \\ & L(e) \leq l(e) \leq u(e) \leq U(e) & e \in E \setminus C \\ & DC\ constraints & \end{aligned}$$

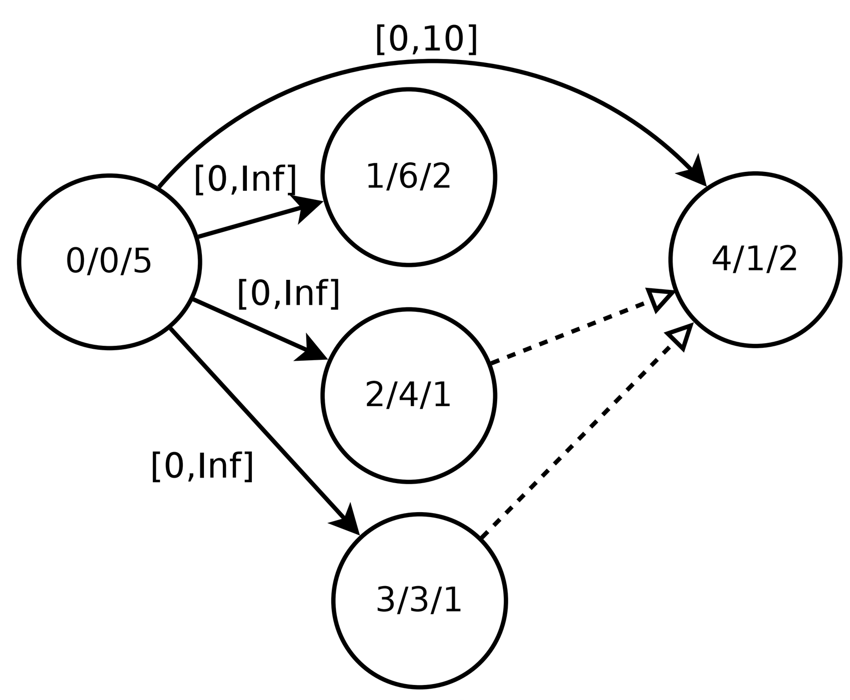
Results







(MD=4,flex=$\frac{5}{10}$,fldt =35.9)



(MD=6,flex=$\frac{4}{10}$,fldt=27.1)

STNU	Topology				Formulation			
Prob Set	Nodes	Links	Ctgs	MaxLags	Vars	Bvars	Constrs	Time(s)
J10	22	62.35	10	3.98	1675.33	546.33	16003.11	18.78
OJ10	22	51.39	10	18	1688.61	559.61	16042.82	29.75
OJ12	26	69.68	12	26	2364.12	807.53	26562.17	191.35
OJ14	30	89.68	14	36	3118.32	1101.32	40817.95	1001.77
OJ20	38	152.12	20	60	4979.12	1816.12	82957.12	5000

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