Formulation of formations: merging of acyclic triangle formations

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Background and motivation

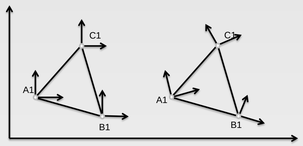
Difference between the two main decentralized formation control approach

➢ Consensus based approach

- Each agent measures their relative positions(displacements) of its neighbouring agents with respect to a global coordinate system.
- The desired formation is specified by the desired displacements between pair of the agents.
- The agents achieve the desired formation by actively controlling the displacements of their neighbouring agents

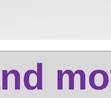
➢ Distance based approach

- Each agent measures the relative positions of its neighbouring agents with respect to its own local coordinate system without any knowledge on a global coordinate system.
- The desired formation is then specified by the desired distances between pair of the agents.
- The agents achieve the desired formation by actively controlling the distances of their neighbouring agents.



Why to combine consensus based formation control law with distance based formation control law?

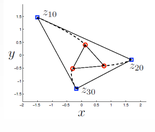
- In distance based approach, agents do not need to share a common orientation comparing with consensus based approach, which means that it is not necessary for agents to equip with orientation sensors.

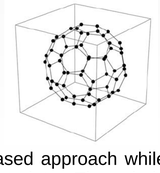
Distance's approach:  Consensus's approach: 

- In distance based approach, agents do not need to consider the collision with other agents while additional algorithms for collision-free need to be designed in consensus based approach.

Distance's approach:  Consensus's approach: 

- The problem of distance based formation control is complicated, current research is mainly focused on small amount of agents and motion in 2D space. But consensus based approach has been approved to be used on forming very complex geometric shape.

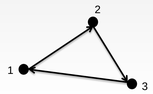
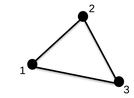
Distance's approach: 

Consensus's approach: 

- "leaders" of sub-formations move according to the consensus based approach while the "followers" are controlled according to the distance constraints. The whole formation can be large and each sub-formation keeps the advantages of the distance based approach .

Architecture of sub-formation

Types of distance based control of triangle formation

Architectures	Information graph	Control law
Distance based undirected formation		$\dot{z}_1 = 0$ $\dot{z}_2 = (z_1 - z_2)(\|z_1 - z_2\|^2 - d_1^2)$ $\dot{z}_3 = (z_1 - z_3)(\|z_1 - z_3\|^2 - d_3^2) + (z_2 - z_3)(\|z_2 - z_3\|^2 - d_2^2)$
Distance based cyclic formation		$z_i = -(d_i - r_i)$
Distance based acyclic formation		$\dot{z}_i = u_i = -\sum_{j\in[\text{edges leaving}]i} \frac{1}{2}(\|e_j\|^2 - d_j)e_j,$

Why to choose the acyclic architecture?

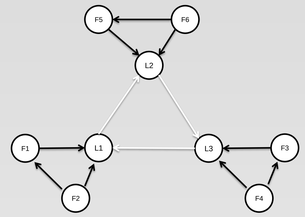
- Leader exists in acyclic architecture in comparison with the undirected architecture and the cyclic architecture, meanwhile the motion of leader does not affect the followers, which means that the stability problem of the whole system can be simplified. We just need to consider whether the desired shape of sub-formation could be reached while the speed of leader converge to 0.

Control law and simulations

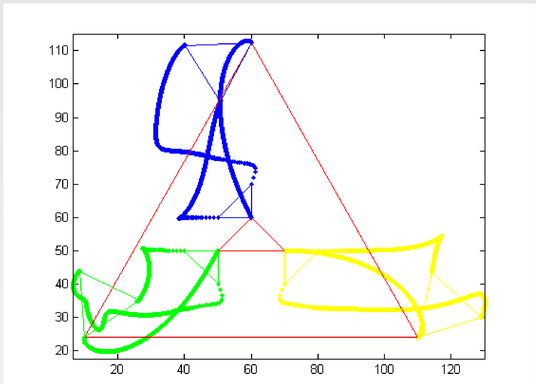
Control law

- Let z1, z2, z3 denote the positions of L1, L2, L3 in a global coordinate system while x1, x2, x3, x4, x5,x6 denote the positions of F1, F2, F3, F4, F5, F6. d1, d2 and d3 are lengths of desired triangle of sub-formation, respectively. Then the control law of the system with the information graph below is:

$$\begin{aligned}
\dot{z}_1 &= -(z_1 - z_2) + \delta_1 \\
\dot{z}_2 &= -(z_2 - z_3) + \delta_2 \\
\dot{z}_3 &= -(z_3 - z_1) + \delta_3 \\
\dot{x}_1 &= (z_1 - x_1)(\|z_1 - x_1\|^2 - d_1^2) \\
\dot{x}_2 &= (x_1 - x_2)(\|x_1 - x_2\|^2 - d_3^2) + (z_1 - x_2)(\|z_1 - x_2\|^2 - d_2^2) \\
\dot{x}_3 &= (z_2 - x_3)(\|z_2 - x_3\|^2 - d_1^2) \\
\dot{x}_4 &= (x_3 - x_4)(\|x_3 - x_4\|^2 - d_2^2) + (z_2 - x_4)(\|z_2 - x_4\|^2 - d_3^2) \\
\dot{x}_5 &= (z_3 - x_5)(\|z_3 - x_5\|^2 - d_1^2) \\
\dot{x}_6 &= (x_5 - x_6)(\|x_5 - x_6\|^2 - d_2^2) + (z_3 - x_6)(\|z_3 - x_6\|^2 - d_3^2)
\end{aligned}$$



Simulations



References

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