Localization Using Bearing-Only Measurements

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Introduction
- Multi-agent formation control has applications in search and rescue, defence, environmental monitoring and more.
- Agents can be aerial drones, ground based robots, mobile sensor networks etc.
- The Defence Science and Technology Organisation (DSTO) are interested in applications for their drones.
- Multiple low-cost agents gives increased reliability, capabilities and effectiveness.
- Control algorithms to move formations require agents to localize each other.
- Localizing: Agent 1 can determine the precise location of Agent 2 at any given time (and vice versa).

Motivation for bearing-only localization
- Standard practise: agents can measure range and bearing of neighbouring agents in a formation.
- Common operational constraint: No GPS and no communication between agents.
- DSTO is interested in removing the range sensor for the following benefits:
  - Cost, payload and space savings.
  - Unlike range sensors, bearing sensors are usually passive and cannot be detected. This is useful in military operations.

Geometric Model
- Two agents are in circular orbit about stationery circle centres.
- Motion in a 2D plane.
- Agents can rotate in either direction.

Solution Part I - Estimating $\omega_2$
From Agent 1’s perspective:
- Apply the Fast Fourier Transform to the measured bearing angle, $\theta(t)$.
  - The 2 largest peaks are the angular velocities.
  - Because we already know $\omega_2$, we arrive at an estimated magnitude of $\omega_2$ but not the direction of rotation.

Solution Part II - System of Equations
The geometric model yields the following equation:

$$r_1(\sin(\omega_2 t + \phi_1) - \tan(\theta(t)) \cos(\omega_2 t + \phi_1)) \approx 0 - \tan(\theta(t)) \omega_2$$

where $r_1$ is the distance between agents and $\phi_1$ is the initial bearing.

Make the following substitutions:
- $a(t) = r_1(\sin(\omega_2 t + \phi_1) - \tan(\theta(t)) \cos(\omega_2 t + \phi_1))$
- $b(t) = -\tan(\theta(t))$
- $c(t) = \cos(\omega_2 t + \phi_1) + \tan(\theta(t)) \sin(\omega_2 t + \phi_1)$
- $d(t) = \sin(\omega_2 t + \phi_1) - \tan(\theta(t)) \cos(\omega_2 t + \phi_1)$

Create an overdetermined system with $n$ equations with $n > 4$.

$$a(t) = \begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \\ a_n(t) \end{bmatrix}, \quad b(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \\ b_n(t) \end{bmatrix}, \quad c(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \\ c_n(t) \end{bmatrix}, \quad d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \\ d_n(t) \end{bmatrix}$$

Step 1: For each trial version of $\omega_2$, find the constants $c_1, c_2, c_3, c_4, c_5, c_6, c_7$ giving the best Least Squares fit, with an associated Least Squares Error.

Step 2: The trial version of $\omega_2$ which minimizes this error is the localization solution. In this example, it is -8 rads⁻¹.

Note: We estimate for both positive and negative values of $\omega_2$ as we do not know the direction of rotation.

Adaptations
Problem: Bearing sensors are often optical cameras with noise in its measurements. Furthermore, we make use of $\tan(\theta)$ in our system of equations. When the measured bearing angle is around 90° or 270° the noise is magnified by the tan function.

Solution: Increasing the number of equations reduces solution error when there is noise. A rotating coordinate frame helps reduce the noise magnification due to the tan function.

Conclusions
- A numerical solution with trade-off between accuracy and solution time.
- The Fourier Transform reduces the solution time for the system of equations by giving an estimated magnitude for the angular velocity.
- Performance is sensitive to noise.
- The basic solution may be adapted depending on the given practical scenario.

Future Work
- Adding translational velocity to centres of agent orbits.
- Achieving velocity consensus for a formation of 3 agents so that the formation moves as a coherent whole.
- Formation shape control of 3 agents.