Robust Formation Control with Distance Mismatch

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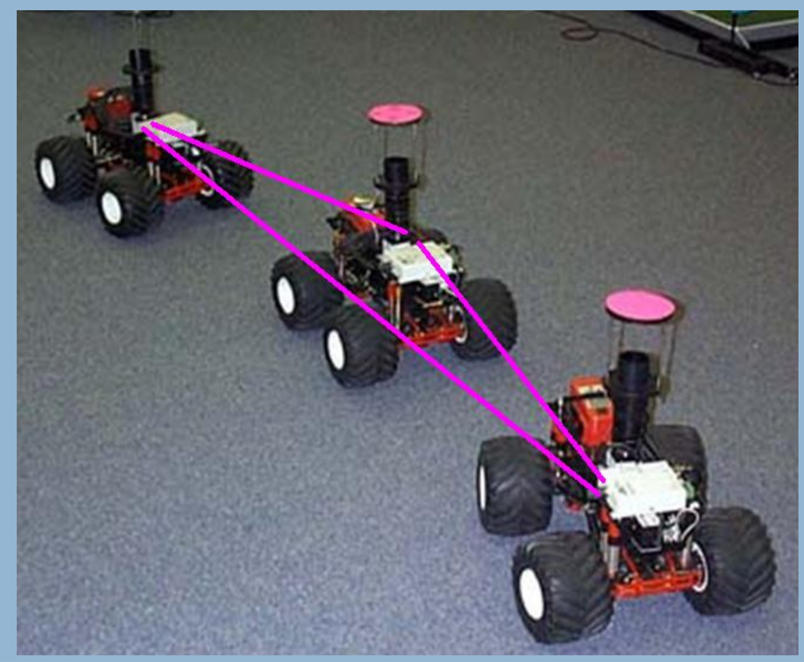
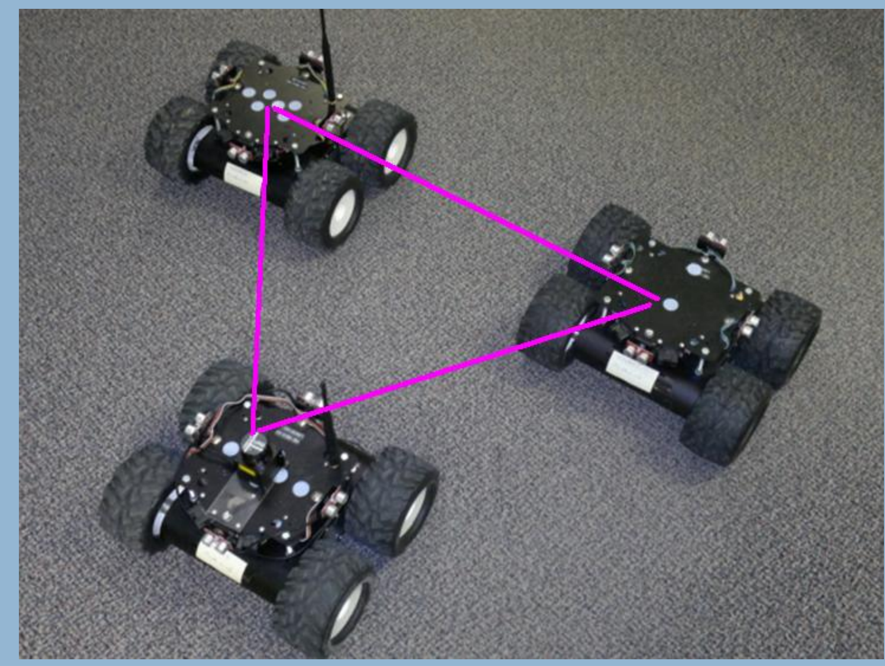
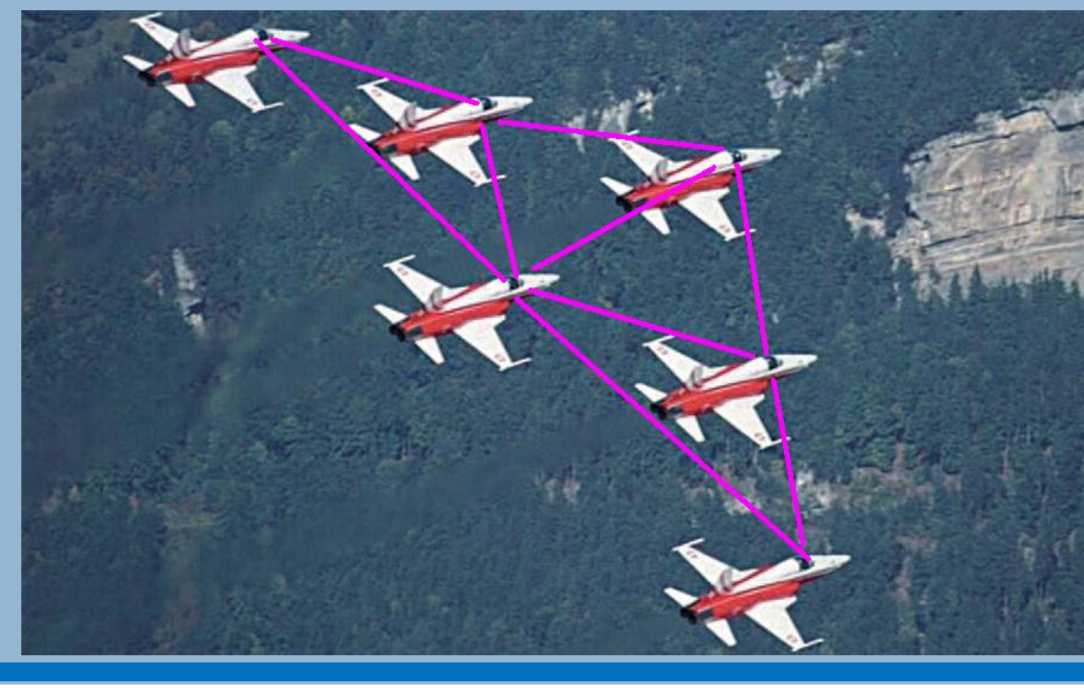
Research and Application Background

Formation shape control for a group of autonomous robots/agents:

- Group of robots/agents need to achieve specific geometrical shapes;
- The controller is distributed, so that only local information is used;
- The shape is specified by a certain set of inter-agent distances;
- Agents should work together to achieve the desired distances.

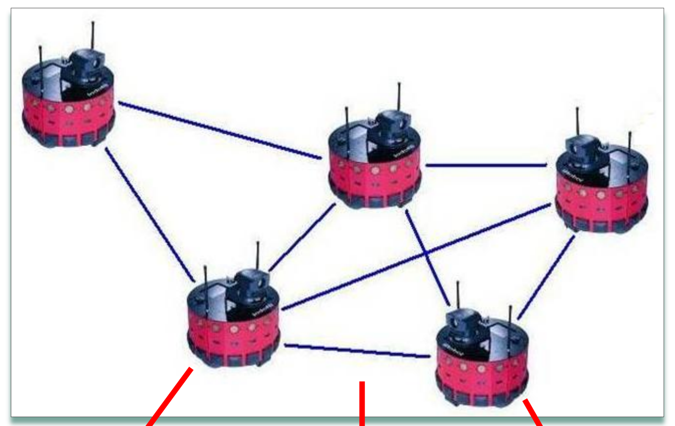


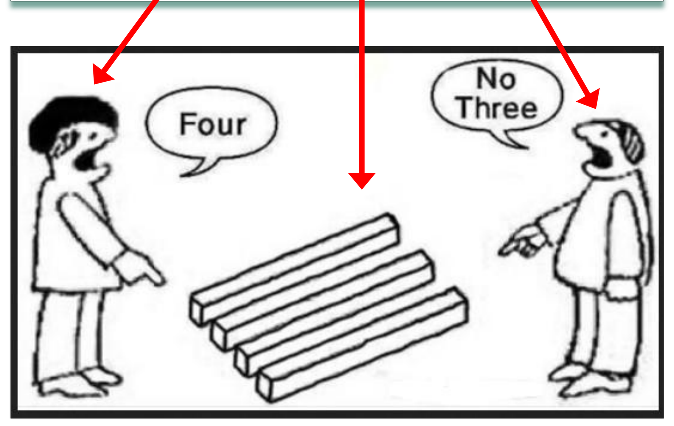




What causes distance mismatch in formation control?

- Rigidity-based formation modeled by undirected graph requires TWO joint agents to maintain each distance.
- Sensors may produce measurement errors, either due to bias or constant drift.
- Some agent pairs may have differing views of the desired distances that they are tasked to maintain.
- Agent pairs may not be aware of the distance mismatch between them.





The distributed controller

Let p_i be the positions of agent i, and d_{kij} be the desired distances between agent i and j that they are required to achieve:

$$V = \frac{1}{4}\sum_{(i,j)\in\mathcal{E}}\left(\|p_i - p_j\|^2 - d_{k_{ij}}^2\right)^2 \quad \xrightarrow{\text{Gradient descent control}} \quad \dot{p} = -\nabla V(p)$$

$$\dot{p}_i = -\sum_{j\in\mathcal{N}_i}(p_i - p_j)e_{k_{ij}}(z)$$

The distance error $e_{k_{ij}} = \|p_i - p_j\|^2 - d_{k_{ij}}^2$

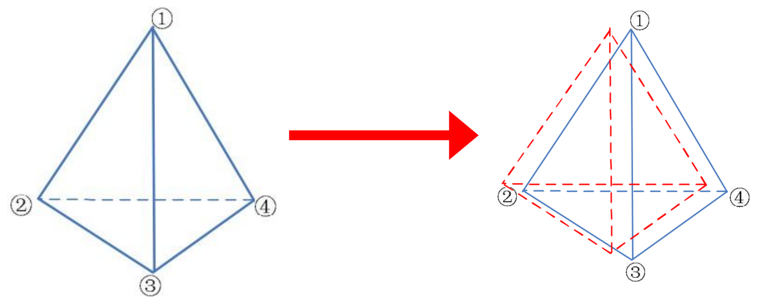
- The potential function measures the total errors between actual distances and desired distances.
- Each agents aims to minimize the potential function, hence the gradient control.

Modified controller, where u_k denotes the mismatch terms →

$$\dot{p}_i = -\sum_{j\in\mathcal{N}_i^-}(p_i - p_j)(e_{k_{ij}}(z) - \mu_{k_{ij}}) + \sum_{j\in\mathcal{N}_i^+}(p_j - p_i)e_{k_{ij}}(z)$$

Good news: a slightly distorted formation shape…

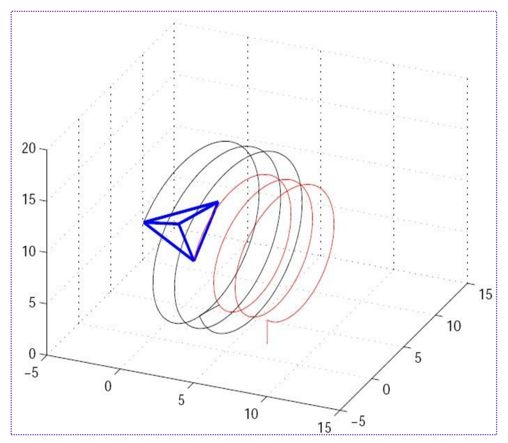
Example: a slightly different 3-D tetrahedron shape.

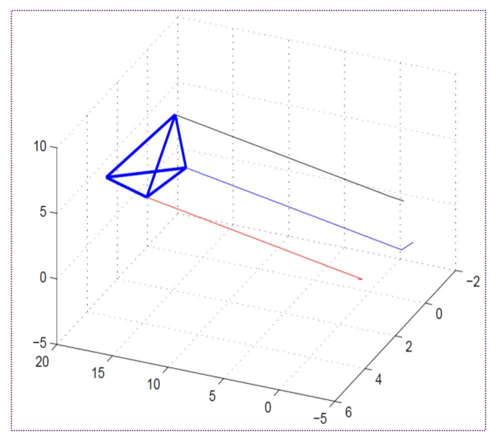


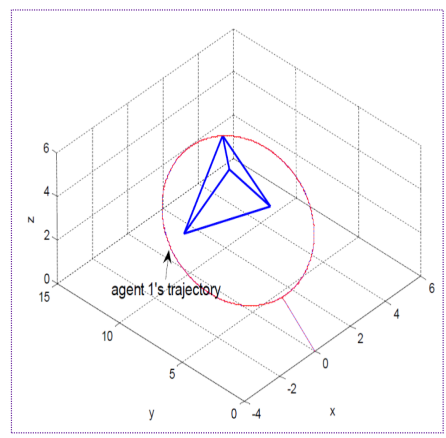
- Distance error will decay exponentially fast to zero;
- Small mismatch will not destroy the formation shape;
- One can still obtain a slightly distorted formation shape;
- However, there are additional motions induced by distance mismatch!

Unexpected motions occur (3-D formations)!

- Rotation and translation of whole formation with axis of rotation also direction of translation—a helical movement!
- Values of agent's speed and angular velocity induced by mismatch are constant.
- Rotation-only or translation-only movement can be observed as special cases but are not generic.

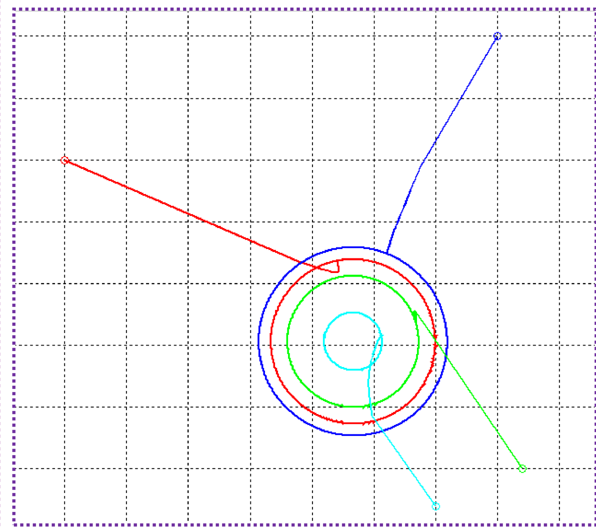






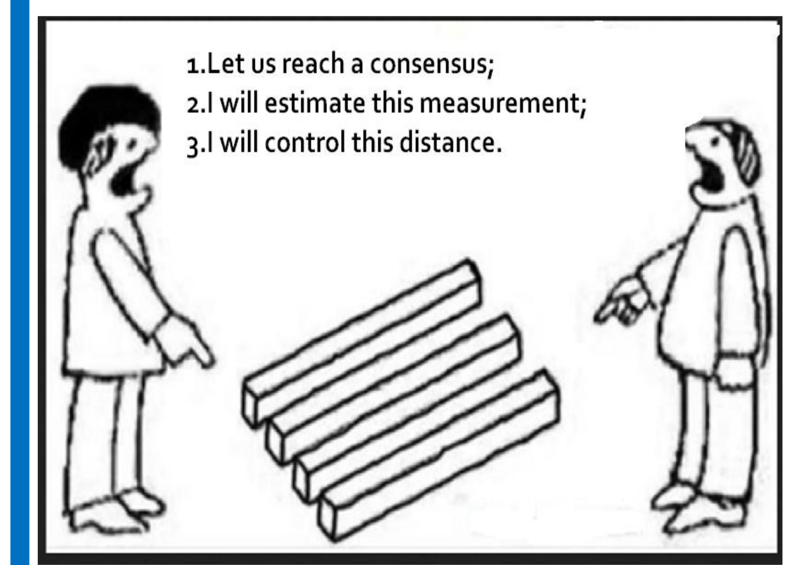
Unexpected motions occur (1-D and 2-D formations)!

- For 1-D formations, it is like a chase and escape scenario. All agents translate in a line.
- For 2-D formations, all the agents generically undergo a circular motion around a common point.
- Translational motion in 2-D case is possible but not generic.



What are the solutions to remove the motions?

- **Directed formation setup:**
  Each edge length is maintained by a single agent.
- **Add a dead zone (the tolerance domain) in the distance errors:**
  The formation stops moving when distance errors enter into the domain.
- **Use additional estimators to reach a consensus :**
  Choose an agent in each edge as the *estimating agent* and design an *adaptive estimation law* to estimate the mismatch.



Further readings:

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[2]. L. Krick, M. E. Broucke, and B. A. Francis, IJC , 2009, 82(3), pp. 423–439.

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[4]. Z. Sun, S. Mou, B. D. O. Anderson and A. S. Morse. AUCC IEEE 2013, pp. 369-374

In collaboration with Shaoshuai Mou and Prof. A. S. Morse, Yale university.

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