Cluster Consensus Control for Generic Linear Agents with Acyclic Partition

By Jiahu Qin; jiahu.qin@anu.edu.au. Supervisor: Changbin (Brad) Yu.

1 Introduction

Very recently, there is an emerging trend to study how an intercon-
ected group may incorporate or evolve into different subgroups, called
clusters. For example,
• in nature, multi-species foraging groups have been observed [1],
such as flocks of bark-foraging birds, in which birds have to coordi-
nate through interactions with peers in their own and other species;
• in social networks, some opinion dynamics models [2] describe
how the agents with bounded confidence levels evolve into differ-
cent clusters, where agents in the same cluster finally hold the same
opinion.

2 Motivation

Definition 1 The $p$-cluster consensus control for multi-agent systems
with partition $(V_1, V_2, ..., V_p)$ is said to be realized if, there exists a
distributed controller $u_i(t)$ such that for any initial states, the states of the
agents satisfy $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0$ when $i = j$ and
$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| \neq 0$ otherwise.

Motivation Example 1: Two clusters $V_1 = \{1, 2, 3\}$ and $V_2 = \{4, 5\}$;
c $> 0$ is the coupling strength among agents within the same cluster.
Each agent takes in the following integrator dynamics:

$$\dot{x}_i = \sum_{j \in N_i} a_{ij}(x_j - x_i),$$

where $a_{ij} > 0$ for the edges within the same cluster, while $a_{ij}$ may be
negative for the edges between two clusters. $G_c$ and $G_0$ are two different interaction topologies, both with clusters $V_1, V_2$.
Initial states for all the agents are randomly chosen from $[-100, 100]$.

3 Directed Acyclic Partition

Definition 2 A directed path is a sequence of edges in a directed
graph of the form $(x_i(t), x_j(t), ...$). When the nodes of the directed
path are distinct except for its end-nodes, the path is called a directed
cycle. A directed graph with no cycles is called directed acyclic graph
or a DAG for short.

3.1 An Interesting Property of DAG

Proposition 1 A DAG can be relabeled such that for any directed
dge, the index of its parent node is smaller than the index of its
child node, and thus its Laplacian matrix is lower triangular.

Figure 1: $G_a$ with $c = 0.01$.

Figure 2: $G_b$ with $c = 0.3$.

where $u_i(t)$ is the distributed control input for agent $i$ which uses only
the state information of its neighboring agents.

Assumption 1 Matrix pair $(A, B)$ is stabilizable.

Remark 1 Under Assumption $1$:

• $A$ is allowed to have exponentially unstable mode;
• Integrator agent dynamics is a special case: $A = 0$ and $B = I_n$.

3.2 Clustering with Acyclic Partition

Given any graph $G$ with partition $\{V_1, ..., V_p\}$, let $G_i, G_i'$ denote
respectively the underlying graphs of node set $V_1, ..., V_p$ in $G$.

Definition 3 $G_i'$, the digraph induced from $G_i$, is defined as follows:

• The node set of $G_i'$ consists of $G_1, ..., G_p$. That is, each $G_i'$ is
dreamed as a node in $G_i$.
• There is an edge $(G_i, G_j) \in G_i'$ from node $G_i$ to node $G_j$ $c_{ij}$
and only if there exist $u \in c(G_i)$ and $v \in c(G_j)$ such that $(u, v) \in c(G)
$.

Lemma 1 For any digraph $G$ with partition $\{V_1, ..., V_p\}$, if the
induced graph $G'$ is acyclic, then the nodes in $G$ can be relabeled such that
the Laplacian matrix associated with $G$ takes in the following form:

$$L_{ij} = \begin{cases} 0 & \text{if } i \neq j, \\ L_{ii} & \text{if } i = j, \end{cases}$$

where $L_{ij}$ is associated with graph $G_i$ and $L_{ij}$ specifies the informa-
tion exchange from cluster $G_i$ to $G_j$.

Assumption 2 For any interaction topology $G$ with partition $\{V_1, ..., V_p\}$,
the induced digraph $G'$ is acyclic.

Assumption 3 (3.4) Each row sum of $L_{ij}$ in $L$ is zero.

4 Distributed Cluster Control: Main Result

To achieve the desired pattern, distributed feedback controller for
each agent is designed as follows:

(1) Select $c = \frac{1}{\epsilon}$, $\epsilon > 0$ such that $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0$;

The distributed feedback controller for agent $i$ is designed as

$$u_i(t) = K \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) + \epsilon(x_i(t) - \bar{x}(t)).$$

where $K$ is the feedback matrix to be designed; $\bar{x}_i > 0$ if agent $i$ is
pinned and $\bar{x}_i = 0$ otherwise.

For each cluster $G_i$, let $c_i$ be the graph comprising $G_i$, the virtual
node $\bar{x}(t)$, and the directed edge from $\bar{x}_i(t)$ to those nodes in $G_i$ which
are pinned.

Theorem 1 For multi-agent systems (1), under Assumptions 1, As-
sumption 2, and Assumption 3, if the agents in each cluster is pinned
such that each $G_i - 1, \ldots , p_i$ has a spanning tree, then the $p$-cluster
consensus control can be realized exponentially fast by using dis-
tributed feedback control law (2).

5 Conclusion

We have studied for the first time the cluster consensus control for
generic linear multi-agent systems and further proved that cluster
consensus control can be realized regardless of the magnitude of the
clustering among agents if the interactions among clusters are in
an acyclic way.

References

[1] A. S. Dolby and T. C. Grubb, "Benefits to satellite members in
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