Measurement Feedback Control of Nonlinear Systems

Tengfei Liu

David J Hill

1 Problem

1.1 Sensors in Control Systems

Sensors to measure physical quantities and convert them into signals which can be read by controllers.

\[ e = z - \tilde{z} \]

Measurement errors are considerable sometimes.

1.2 Examples of Measurements in Control Systems

Example 1: Noisy Measurement

Properties:
- Non-differentiable (non-smooth);
- Unbounded.

Example 2: Logarithmic Quantizer

Properties:
- Non-differentiable (non-smooth);
- Bounded.

Example 3: Dynamic Quantization

Properties:
- Non-differentiable (non-smooth);
- Dynamical.

2 Tools

Two main tools used in measurement feedback control design:
- Small-gain theorem;
- Set-valued control.

2.1 Small-gain Theorem for Dynamical Networks

A dynamical network of ISS systems is ISS, if the composition of the ISS gains along every simple cycle is less than 1 (identity).

2.2 Set-Valued Control and Differential Inclusions

Consider a nonlinear control system:
\[ x = f(x, u) \]

Given set-valued control
\[ u \in S_{\mu}(x, \omega) \]

Define set-valued map
\[ F(x, \omega) = \{ f(x, u) : u \in S_{\mu}(x, \omega) \} \]

We get differential inclusion
\[ \dot{x} \in F(x, \omega) \]

Theory of differential inclusions:
- Solution;
- Stability.

3 Theoretical Achievement

We developed measurement feedback controller for nonlinear uncertain systems:
\[ x_i = x_i(t) + f_i(x_i, \ldots, x_i, \omega_i) \]
\[ \dot{\omega}_i = g_i(x_i, \ldots, x_i, \omega_i) \quad 1 \leq i \leq n \]

The measurement errors may be:
- Discontinuous (non-smooth);
- Unbounded;
- Dynamical.

4 Synchronization with Measurement

Master System:
\[ x_1 = y_1 \]
\[ \dot{y}_1 = -x_1 + (1 - x_1^2)y_1 \]

Slave System:
\[ x_2 = y_2 \]
\[ \dot{y}_2 = -x_2 + (1 - x_2^2)y_2 + u \]

Useful for:
- Stability analysis of large-scale systems;
- Control design of nonlinear systems with interconnections and loops.

Figure 1: A control system.

Figure 2: Sensor—True value and measured value.

Figure 3: Non-differentiable measurement.

Figure 4: Logarithmic quantizer.

Figure 5: Dynamic quantization.

Figure 6: Synchronization performance—1.

Figure 7: Synchronization performance—2.