Synchronization of Complex Dynamical Networks by Network Control

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1 A Complex Networks World

Nowadays, we are living in a world which is full of complex dynamical networks (CDNs). These networks are not only supporting massive information transmission, but also becoming a part of our life. They bring us tremendous convenience and meanwhile provide us greater challenges than ever before.

1.1 What Is a CDN?

A complex dynamical network (CDN) is a large set of interconnected communicating and interacting nodes where a node is a fundamental unit having specific contents and exhibiting dynamical behaviour.

1.2 The Complexities of CDNs

In the study of CDNs, the main difficulties come from the complexities of CDNs which are listed as follows:

- **Structural complexity:** the wiring diagram could be an intricate tangle and change over time, meanwhile the links between nodes could have different weights, directions and signs (see Fig. 1.a);
- **Dynamical complexity:** the node could be nonlinear dynamical systems, and there could be many different kinds of nodes (see Fig. 1.b);
- **Meta-complication:** the various complications can influence each other.

1.3 Synchronization of CDNs

Synchronization phenomenon are ubiquitous in nature (see Fig. 3.) and play a very important role in many different contexts as biology, ecology, climatology, sociology, technology, or even in arts.

3.1 What Is a Network Controller?

A network controller (NC) is a distributed controller by adding some

3.2 Switching Controller Design

Sometimes a single NC can not synchronize the network under the energy constraint, and sometimes due to the limitation of numerical algorithms, it is hard to find such a NC which does exist. Switching control technique provides an alternative way to deal with these problems. With the help of the tools listed below

- Single Lyapunov function method,
- Inverse Lyapunov function theorem,
- Algebraic graph theory,
- Convex optimization theory,

we can

- Identify the synchronized switching signal
- By solving a convex optimization problem;
- Design candidate network controllers
- By solving mixed-integer nonlinear optimization problems.

4 Network Controller Design

Based on the MSF method, we study the local synchronization of CDNs with identical nodes by designing a NC or a switching NC under a certain energy constraint.

4.1 Single Network Controller Design

By combining the following theories with MSF method:

- Algebraic graph theory,
- Optimization theory,

we can overcome the difficulties caused by the complexity of the CDN and the energy constraint, and convert the design problem into solving a mixed-integer nonlinear optimization problem.

Figure 5 is an example with a 10 nodes network whose nodes are 3-dimensional identical linear systems to illustrate effectiveness of the design method.

5 Future Works

- Global results;
- Time-delay in the control path;
- More general switching controllers;
- Non-identical nodes.